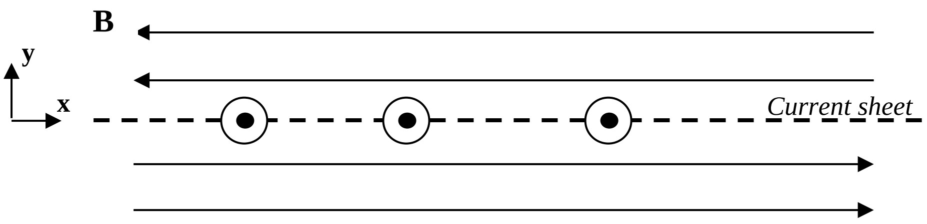
Solutions, Tutorial 5

1.

Define the coordinate system as below, to be compatible with the expression we derived for an infinite current sheet.



Then

$$\iint j_z dxdy = -\frac{1}{\mu_0}\int \frac{\partial B_x}{\partial y} dy \cdot \Delta x = \frac{1}{\mu_0}\Delta B_x \cdot \Delta x = \frac{1}{4\pi \cdot 10^{-7}} \cdot 10 \cdot 10^{-9} \cdot 1.5 \cdot 10^{11} = 1.2\ \text{GA}$$

2.

a)

$$\mathbf{v} = \frac{\mathbf{E}\times\mathbf{B}}{B^2}$$

The parallel electric field in the ionosphere is very small due to the high field-aligned conductivity, so the electric and magnetic fields are parallel to each other. Therefore

$$\text{v} = \frac{E}{B} \quad \Rightarrow$$
$$E = vB$$

From the figure, I get v = 230 m/s. The magnetic field strength we get from the dipole strength with $r = R_E +$ 300 km, and $\theta = 90°-65° = 25°$. Then

v = 6/11·500 m/s = 273 m/s

$$B = \sqrt{B_r^2 + B_\theta^2} = \sqrt{B_P^2\left(\frac{R_E}{r}\right)^6 \cos^2\theta + \left(\frac{B_P}{2}\right)^2\left(\frac{R_E}{r}\right)^6 \sin^2\theta} =$$

$$B_P\left(\frac{R_E}{r}\right)^3\sqrt{\cos^2\theta + \frac{\sin^2\theta}{4}} = B_P\left(\frac{R_E}{R_E + 300\ \text{km}}\right)^3\sqrt{\cos^2 25^\circ + \frac{\sin^2 25^\circ}{4}} =$$

= 50 266 nT.

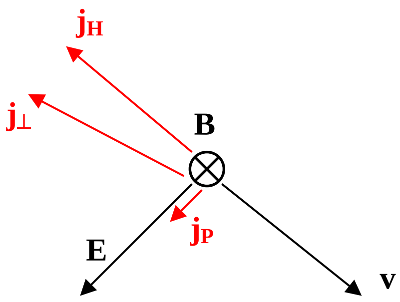
Then

$E = vB$ = 13.7 mV/m.

b)

Using solar maximum values at 100 km altitude, I get (night side values)
$\sigma_P = 8 \cdot 10^{-7}$ S/m
$\sigma_H = 7 \cdot 10^{-6}$ S/m
Then
$j_P = \sigma_P E = 7.3 \cdot 10^{-9}$ A/m^2
$j_H = \sigma_H E = 6.4 \cdot 10^{-8}$ A/m^2

b)



Taking the E region altitude to be 100 km (see Fälthammar Fig 3.2.1 b, p. 28) and reading off the night side conductivities from Fälthammar p. 31 (I used solar maximum values), we get

$j_P = \sigma_P E = 8 \cdot 10^{-7} \cdot 11.6 \cdot 10^{-3} = 9.3$ nA/m^2

$j_H = \sigma_H E = 7 \cdot 10^{-6} \cdot 11.6 \cdot 10^{-3} = 81$ nA/m^2.

3.

a)

The strongest magnetic field the proton will encounter is 31 μT. The gyro radius ρ is given by

$$\rho = \frac{p_\perp}{qB}$$

With the high energies associated with the cosmic ray particles, we need to use the relativistic expression for the momentum (Fälthammar p. 144):

$pc = \sqrt{E^2 - m_0^2 c^4}$, where m_0 is the proton rest mass (1.67×10^{-27} kg).

Then

$$\rho = \frac{\sqrt{E^2 - m_0^2 c^4}}{cqB}$$

$$\Rightarrow$$

$$E = \sqrt{(\rho q B)^2 c^2 + m_0^2 c^4} =$$

$$= \sqrt{(10 R_E)^2 \cdot (1.6 \cdot 10^{-19})^2 \cdot (31 \cdot 10^{-6})^2 \cdot (3 \cdot 10^8)^2 + (1.67 \cdot 10^{-27})^2 (3 \cdot 10^8)^4} =$$

$$= \sqrt{9.0 \cdot 10^{-15} + 2.3 \cdot 10^{-20}} = 9.5 \cdot 10^{-8}\ \mathrm{J} = 5.9 \cdot 10^{11}\ \mathrm{eV}$$

Note that the rest energy of the proton (1 GeV) is negligible compared to this.

b)

$$\rho = \frac{p_\perp}{qB} = L$$

$$\Rightarrow$$

$$B = \frac{p_\perp}{qL} = \frac{\sqrt{E^2 - m_0^2 c^4}}{eLc} = \frac{\sqrt{10^{16} \cdot (1.6 \cdot 10^{-19})^2}}{1.6 \cdot 10^{-19} \cdot 100 \cdot 3 \cdot 10^8} = \frac{10^8}{100 \cdot 3 \cdot 10^8} = 3.3\ \mathrm{mT}$$

For comparison: the magnetic field in the centre of a circular loop current is

$$B = \frac{\mu_0 I}{2r}$$

$$\Rightarrow$$

$$I = \frac{2Br}{\mu_0} = \frac{2 \cdot 0.003 \cdot 100}{4\pi \cdot 10^{-7}} = 0.5\ \mathrm{MA}$$

4.

The Strömgren radius is

$$R_S = \left(\frac{3 N_{UV}}{4\pi \alpha_H n_H^2} \right)^{1/3} \Rightarrow$$

$$R_s^3 = \frac{3 N_{UV}}{4\pi \alpha_H n_H^2} \Rightarrow$$

$$\alpha_H = \frac{3 N_{UV}}{4\pi n_H^2 R_s^3} \Rightarrow$$

$$2 \cdot 10^{-16} \cdot T_e^{-\frac{3}{4}} = \frac{3N_{UV}}{4\pi n_H^2 R_s^3} \Longrightarrow$$

$$T_e = \left(\frac{3 \cdot N_{UV}}{4\pi n_H^2 R_s^3 \cdot 2 \cdot 10^{-16}}\right)^{-\frac{4}{3}} = \left(\frac{3 \cdot 10^{48}}{4\pi (10^8)^2 \cdot (7.5 \cdot 9.5 \cdot 10^{15})^3 \cdot 2 \cdot 10^{-16}}\right)^{-\frac{4}{3}}$$

$$= 43800 \text{ K}$$